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Economic dispatch of thermal generation

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ABSTRACT

This paper deals with the formulation and solution of the optimal economic operation problem of thermal power systems over short term periods. An appropriate mathematical model and a method of computer solution based on the first order gradient technique are applied to the system with and without consideration of the transmission losses. To obtain more economic generation an economic dispatch, in which new sets of B-coefficients of transmission losses formula are calculated according to the daily variations of load on the system, will be presented.

1. INTRODUCTION

Economic power system operation deals with the means and techniques for achieving minimum operating cost to supply a given predicted load demand. There is a need to expand the limited economic optimization problem to incorporate constraints on system operation to ensure the security of the system [1 to 4].

The original problem of economic operation of thermal power systems can be successfully solved by numerous methods. Gradient methods have one very strong desirable characteristic. That is, the gradient search technique which always start off with a feasible solution and search for the optimum along a trajectory that maintains a feasible solution at all times [5].

In this paper an economic dispatch will be presented, in which new sets of B-coefficients of transmission losses formula are calculated according to the daily variations of load on the system, to achieve more economic generation. The proposed dispatch is based on the first order gradient method. It is applied to a power system, with and without the transmission losses taken into consideration, to achieve minimum operating fuel costs. The transmission losses of the power system are calculated by using the loss-formula coefficients.

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2. ECONOMIC DISPATCH OF THERMAL GENERATION

A thermal power system consists of (N) thermal generating units is connected to an equivalent load bus through a transmission network. The optimization time period will be divided into (n) time intervals. The total generating cost of the system (F_T) is equal to the sum of the generating costs of each unit (F_i) and must be a minimum value in each time interval [7 to 9], as given by

$$F_T = \sum_{i=1}^{N} F_i(P_i) = \text{Minimum} \qquad (1)$$

and

$$F_i(P_i) = A_i\ P_i^2 + B_i\ P_i + C_i \qquad (2)$$

where A_i, B_i and C_i are the cost constants of unit (i). The sum of the outputs must be equal to the power demanded by the load (P_R) in addition to the transmission losses (P_L), as shown in the following constraint equation

$$\sum_{i=1}^{N} P_i = P_L + P_R \qquad (3)$$

The power of each unit (P_i) must be greater than or equal to the minimum power permitted and also must be less than or equal to the maximum power permitted on that particular unit. These inequalities are thus .

$$P_i^{min} \le P_i \le P_i^{max} \qquad (4)$$

The constraints , which are given in eqns. (3) and (4) must be satisfied in each time interval.

Gradient method start off with a solution in which all the constraint conditions are met (feasible solution) , and search for the optimum solution along a trajectory that maintains a feasible solution at all times. The gradient search method is allowed to continue until the search procedure can find no additional significant gain in the objective function or until the number of iterations has exceeded some reasonable high value [5].

By allowing each of the powers of eqn. (3) to be perturbed some small amount, the constraint equation becomes as follows :

$$\sum_{i=1}^{N} \Delta P_i = \Delta P_L = \sum_{i=1}^{N} (d P_L / d P_i) \Delta P_i \qquad (5)$$

Using Taylor series expansion in eqn. (1) and neglecting the second and higher order terms, the cost equation (objective function) can be obtained as

$$F_T + \Delta F_T = F_1(P_1) + F_2(P_2) + \ldots\ldots + F_N(P_N)$$

$$+ (d\,F_1 / d\,P_1)\,\Delta P_1 + \ldots\ldots + (d\,F_N / d\,P_N)\,\Delta P_N \qquad (6)$$

Thus, the difference in the total cost is obtained as

$$\Delta F_T = \sum_{i=1}^{N} (d\,F_i / d\,P_i)\,\Delta P_i \qquad (7)$$

One unit must be selected as a dependent unit (unit number x), then eqn. (5) can be rewritten as

$$\sum_{i=1}^{N} \Delta P_i + \Delta P_x = \sum_{i=1}^{N} (d\,P_L / d\,P_i)\,\Delta P_i + (d\,P_L / d\,P_x)\,\Delta P_x \quad , (i \neq x) \qquad (8)$$

From eqn. (8) , ΔP_x can be obtained as

$$\Delta P_x = \frac{\sum_{i=1}^{N} [\,\Delta P_i - (d\,P_L / d\,P_i)\,\Delta P_i\,]}{[\,(d\,P_L / d\,P_x) - 1\,]} \quad , (i \neq x) \qquad (9)$$

ΔP_x is the change in output power of the dependent unit in terms of the change in output powers of the remaining (N − 1) units.
Eqn. (7), then, becomes

$$\Delta F_T = \sum_{i=1}^{N} [\,(d\,F_i / d\,P_i) + Z_i\,(d\,F_x / d\,P_x)\,]\,\Delta P_i \quad , (i \neq x) \qquad (10)$$

where $Z_i = [1 - (d\,P_L / d\,P_i)] \,/\, [(d\,P_L / d\,P_x) - 1] \quad , (i \neq x) \qquad (11)$

When the transmission loss power (P_L) is calculated using the loss-formula coefficients, then

$$(d\,P_L / d\,P_i) = 2 \sum_{j=1}^{N} B_{ij}\,P_j \quad , (i \neq x) \qquad (12)$$

and

$$(d\,P_L / d\,P_x) = 2 \sum_{j=1}^{N} B_{xj}\,P_j \qquad (13)$$

The general form of transmission loss (P_L) as a function of unit generation is given by

$$P_L = P^T\,[\,B\,]\,P + P^T\,B_o + B_{oo} \qquad (14)$$

Where P is the vector of generator bus power (MW), [B] is the square matrix having the same dimension as P, B_o is the vector of the same length as P and B_{oo} is constant.

Eqn. (14) can be rewritten, after neglecting the second and third terms, as follows

$$P_L = \sum_i \sum_j P_i B_{ij} P_j \tag{15}$$

The B-matrix loss formula is developed using a series of transformations on the full-impedance matrix of the transmission system network [1]. Two assumptions are built into the loss formula derivation. The first assumes that each generator's reactive power (Q_i) is a linear function of its real power output (P_i). That is,

$$Q_i = S_i P_i \tag{16}$$

The second assumption built into the calculation concerns the reference-bus voltage. This voltage is assumed known and constant as loading varies on the system. The final form of the B-coefficients of the transmission losses is, [6].

$$B_{ij} = \frac{R^3_{ij} + R^3_{ji}}{2|V_i|\,|V_j|} [\,(\cos\theta_i + S_i \sin\theta_i)(\cos\theta_j + S_j \sin\theta_j) + (\sin\theta_i - S_i \cos\theta_i)(\sin\theta_j - S_j \cos\theta_j)\,] \tag{17}$$

Where R^3 are the elements from reference frame 3, $[Z^3]$, S_i, S_j are as defined in eqn. (16), and $|V_i|$, $|V_j|$, θ_i and θ_j are the bus voltages and phase angles from the base-load flow.

The following algorithm is used to obtain the proposed economic dispatch :-

1. Read input data (number of generating units, number of time intervals, line and bus data, cost functions and power limits,...).
2. Define the time interval and the corresponding demanded load.
3. Calculate matrix of B-coefficients as follows :
   - calculate the open circuit impedance matrix of reference frame 1,
   - calculate bus admittance matrix,
   - use Gauss-Seidel method of load flow study to determine line flows, bus voltages, active and reactive power generated,..

- calculate the impedances of reference frames 2 and 3, then define the real terms of the impedance of reference frame 3,
- calculate the B-coefficients as given in eqn.(17).

4. Calculate the transmission losses as given in eqn.(15).
5. Assume a feasible solution of P_i, i = 1 to N, satisfying eqn.(3), (k = 1 , k is the iteration number).
6. Select the dependent unit x and its power P_x.
7. Calculate $F_x' = d F_x / d P_x$, $F_i' = d F_i / d P_i$ and Z_i as given in eqn.(11), i = 1 to N and i ≠ x .
8. Calculate $(F_i' + Z_i F_x')$, i = 1 to N and i ≠ x , and define the order I of the greatest value.
9. Check about the constraint in eqn.(4) for unit I and adjust the value of ΔP_I and calculate the new value of P_I .
10. Calculate the value of ΔP_x from eqn.(9) and find the new value of P_x . (k = k + 1).
11. Check the constraint in eqn.(4) for the dependent unit x. If this constraint is not satisfied, select a new dependent unit and go to step (7).
12. Calculate F_i (i = 1 to N), F_T and ΔF_T (where ΔF_T is the difference between F_T in this iteration and the corresponding value in the previous iteration).
13. if ΔF_T is within the tolerance value, print the results (each unit generation, transmission losses, each unit generation cost and total generation costs), if not go to step (3).
14. For the next time interval go to step (2).

3. RESULTS

Tables (1) and (2) list the input line data and bus data of the used sample power system [6]. The cost function ,the power limits of each generating unit (the base is 100 MVA) and also the data of daily load curve are given in Tables (3) and (4), respectively.

Table (1) Input line data.

Line No.	From bus	To bus	R (p.u.)	X (p.u.)
1	1	4	0.0570	0.0845
2	1	5	0.0133	0.0360
3	4	5	0.0194	0.0625
4	2	5	0.0173	0.0560
5	2	3	0.0319	0.1750
6	2	6	0.0300	0.1500

Table (2) Input bus data.

Bus No.	Assumed bus voltage (p.u.)	Generation P (p.u.)	Generation Q (p.u.)	Load P (p.u.)	Load Q (p.u.)
1	1.05 + j 0.00 (specified)	-	-	-	-
2	-	0.30	0.05	-	-
3	-	0.10	0.05	-	-
4	-	-	-	0.40	0.05
5	-	-	-	0.30	0.05
6	-	-	-	0.15	0.03

Table (3) Cost functions and power limits data.

Unit No.	A ($/Mw^2 h)	B ($/Mw h)	C ($/h)	P^{max} (Mw)	P^{min} (Mw)
1	0.001562	7.920	561	600	150
2	0.001940	7.850	310	400	100
3	0.004820	7.970	78	200	50

Table (4) Daily load curve data :

Inter. No.	1	2	3	4	5	6	7	8	9	10
Time	0-2	2-4	4-6	6-8	8-10	10-14	14-16	16-18	18-22	22-24
Hours No.	2	2	2	2	2	4	2	2	4	2
Load (MW)	500	350	450	550	700	950	600	1050	1150	850

The mathematical model is formed as mentioned in section (2) to obtain the optimum scheduling of the powers with minimum generation costs at each time interval. The proposed economic dispatch is presented such that the transmission losses are taken in consideration and new sets of the B-coefficients are calculated according to the daily variations of load on the system.

The obtained values of generation of each unit, transmission losses, total generation and generation cost of each unit in addition to the total generation cost at each time interval are tabulated, respectively, in Table (5). The obtained optimum hourly load sharing is shown in Fig.(1).

Table (5) Power generation (Mw), losses (Mw), and generation costs ($), (when the B-Coefficients are varied).

Int.	F_1	F_2	F_3	F_L	P_T	F_1	F_2	F_3	F_T
1	234.52	203.60	64.16	2.29	502.3	5008.6	3977.5	1218.5	10204.6
2	189.23	109.92	51.87	1.03	351.0	4231.3	2392.7	1008.8	7632.8
3	207.66	183.98	59.73	1.38	451.4	4546.2	3639.9	1142.5	9328.6
4	258.27	227.50	66.90	2.68	552.7	5421.5	4392.6	1265.6	11079.7
5	323.66	273.72	105.81	3.20	703.2	6576.1	5208.1	1950.6	13734.8
6	456.29	375.00	123.40	4.71	954.7	18000.4	14106.2	4539.8	36646.5
7	287.37	243.09	72.82	3.10	603.1	5932.1	4665.8	1367.9	11965.8
8	526.99	388.65	142.28	7.93	1057.9	10337.1	7307.9	2619.2	20264.3
9	594.16	399.98	164.88	9.03	1159.0	23272.9	15040.9	6092.6	44406.4
10	392.89	300.00	160.50	3.41	853.4	7827.6	5679.2	2962.8	16469.6
Total fuel cost over the whole period =									181733.1

When the B-coefficients are calculated at the first load value and maintained as fixed values for all other load variations, the corresponding obtained values are given in table (6).

Table (6) Power generation (Mw), losses (Mw), and generation costs ($), (when the B-Coefficients are fixed).

Int.	F_1	F_2	F_3	F_L	F_T	F_1	F_2	F_3	F_T
1	235.08	203.52	63.41	2.02	502.0	5018.4	3976.0	1205.5	10199.9
2	160.37	140.64	50.00	1.03	351.0	3742.7	2904.9	977.1	7624.8
3	210.02	181.90	59.71	1.64	451.6	[illegible]	3604.2	1142.2	9333.1
4	258.77	222.36	71.31	2.46	552.5	5430.2	4303.0	1341.8	11074.9
5	331.04	279.48	93.49	4.02	704.0	6708.1	5310.9	1730.5	13749.6
6	452.58	367.58	137.36	7.57	957.5	17861.4	13830.4	6054.8	36746.8
7	282.28	238.53	82.12	2.55	602.9	5842.3	4585.7	1530.1	11958.2
8	528.26	385.20	142.53	8.99	1058.9	10361.4	7299.8	2623.9	20284.8
9	593.72	399.69	167.53	10.95	1160.9	23255.8	15030.2	6193.9	44490.0
10	404.62	335.87	115.45	5.96	855.9	8042.7	6330.9	2124.9	16498.6
Total fuel cost over the whole period =									181950.7

Fig.(2) shows the calculated transmission losses P_L of the power system when the B-coefficients are calculated at the first load value and maintained fixed as constant values for all other load variations and when these coefficients are recalculated according to the variations of the load versus the total load demand, respectively.

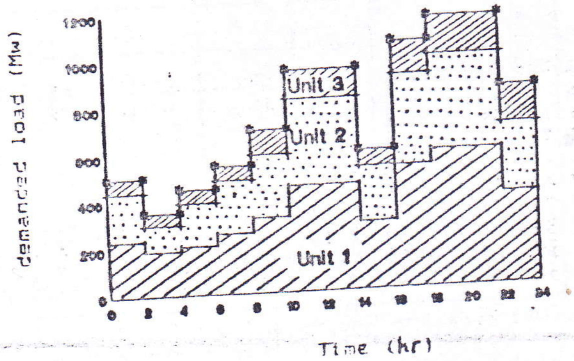

Fig.(1) Cptimum load sharing on the daily load curve.

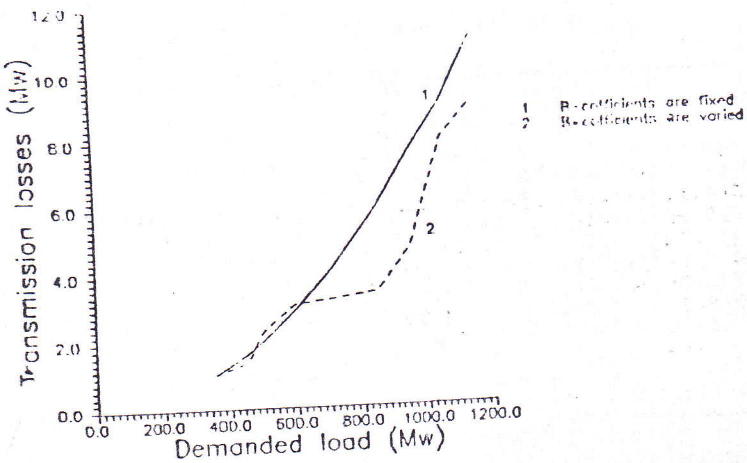

Fig.(2) Transmission losses versus demanded load.

When the transmission losses are not taken into account, Table (7) lists the corresponding obtained values.

Table (7) Power generation (Mw) and costs of generation ($).
(when the transmission losses are not taken into consideration).

Int.	P_1	P_2	P_3	P_L	P_T	F_1	F_2	F_3	F_T
1	230.00	193.33	67.67	–	500.	4930.5	3800.4	1434.7	10165.5
2	160.00	140.00	50.00	–	350.	3736.4	2894.0	977.1	7607 5
3	206.67	185.00	58.33	–	450.	4529.0	3657.3	1118.6	9304.9
4	255.00	222.50	72.50	–	550.	5364.3	4705.3	1362.3	11031.9
5	335.94	264.73	99.32	–	700.	6796.0	5048.2	1834.3	13682.6
6	440.00	376.67	133.33	–	950.	17392.8	14168.3	4905.4	36466.5
7	276.15	238.23	85.62	–	600.	5734.6	4580.3	1591.4	11906.3
8	494.00	400.00	156.00	–	1050.	9709.3	7520.8	2877.2	20107.4
9	570.00	400.00	180.00	–	1150.	22331.3	15041.6	6675.1	44048.2
10	393.33	332.45	124.17	–	850.	7835.7	6269.1	2283.9	16388.8
Total fuel cost over the whole period =									180709.6

The optimum power generation of the units versus the total demanded load is shown in Fig.(3). Fig.(4) shows the optimum fuel cost per hour for each unit versus the total demanded load.

Table (8) Comparison between economic dispatch results with fixed and varied B-coefficients.

	Economic dispatch	
	with fixed B-coff's	with varied B-coeff's
Total generation cost ($)	181950.7	181733.1
Total trans. losses (Mwh)	132.024	104.992
Saving in generation costs ($)	–	217.6
Saving in trans. losses (Mw h)	–	27.032
Excess in computing time	–	35 %

The total generation costs and the total transmission losses (Mwh) in the total optimization time period with fixed varied B-coefficients are listed in Table (8). The saving in the total generation costs and

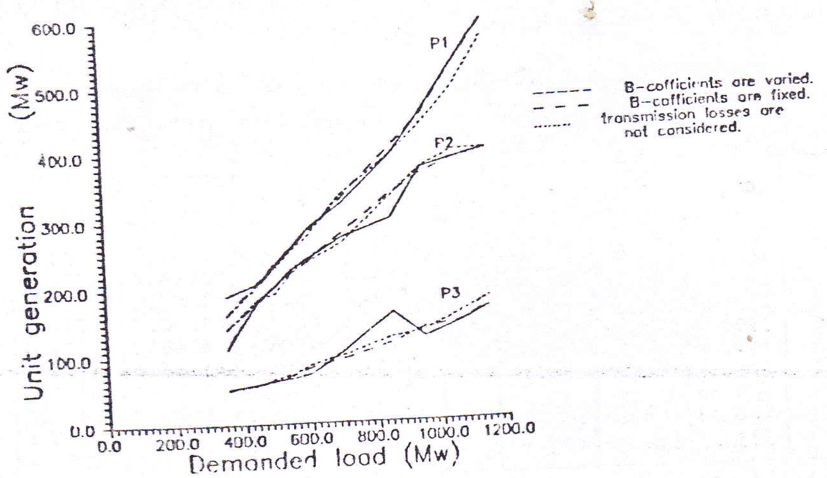

Fig.(3) Optimum power generation of the units versus demanded load.

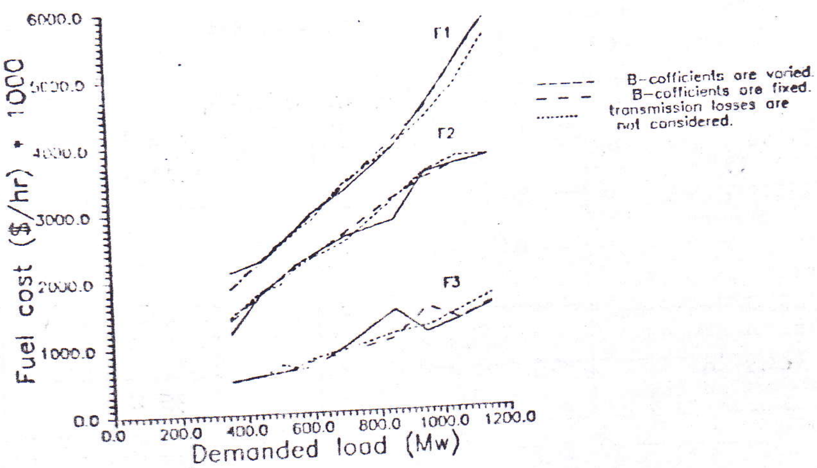

Fig.(4) Optimum fuel cost per hr for each unit versus demanded load.

the total transmission losses (Mwh) during the optimization period are also tabulated in Table (8). This saving results due to the calculation of new sets of B-coefficients according to the variations of the load. THe excess in the computing time for the economic dispatch with the two cases of the B-coefficients is also listed in the same table.

4. CONCLUSIONS

An economic dispatch was presented to obtain more economic generation for thermal power systems. In the proposed dispatch new sets of B-coefficients of transmission losses formula are calculated according to the daily variations of load on the system. The recalculation of the B-coefficients according the load variation takes more computing time but gives accurate sharing of the generation units.
The corresponding generation costs and transmission losses (Mwh) during the optimization period will be less than that when the coefficients are calculated at the first load value and maintained constant with daily load variations.

5. REFERENCES

1. Kirchmayer, L.K., " Economic operation of power systems ", John Wiley and Sons ,New York, 1958.
2. Edelmann, H., " Optimaler Verbundbetrieb unter besondern Nebenbedingungen", Electrizitatswirtsnaft, 60. Jahrgang, Heft 3, Februar 1961.
3. Agarwal, S.K., Nagrath, L.J., " Optimal scheduling of hydrothermal systems ", Proc. IEE, Vol. 119, No. 2, February, 1972, pp. 169-173.
4. Saha, T.N., Khaparde, S.A., " An application of a direct method to the optimal scheduling of hydrothermal system ", IEEE Trans. on PAS , Vol. PAS - 97, No. 3, May / June, 1978, pp. 977 - 983.
5. Allen, J. Wood and B.F. Wollenberg , " Power generation, operation and control ", John Wiley and Sons, New York, 1984.
6. Murty, P.S.R., " Power system operation and control ", Mc Graw Hill Book Company, New Delhi , 1984.
7. Mohamed-Nor, Kh., Abdel Rashid, A.H., " Efficient economic dispatch algorithm for thermal unit commitment ", IEE Proceedings - C, Vol. 138, No. 3, May, 1991, pp. 213 - 217.
8. Abdel Maksoud, S.M., " Short-term optimal economic operation of hydrothermal power system by gradient method ", M.Sc. thesis, Faculty of Eng. (Shoubra), Zagazig University, Cairo, Egypt, 1992.
9. Salama, M.M., " Dynamic model for thermal power systems ", Journal of Eng. and App. Science, Vol. 40, No. 5, October, 1993, pp. 1007 - 1018, Faculty of Eng., Cairo University, Egypt.

أسلوب إقتصادي للتوليد الحراري

يقوم هذا البحث بتكوين وحل مشكلة التشغيل الاقتصادي الأمثل لأنظمة القوي الحراريه على الفترات الزمنية القصيره.
وتم عمل النموذج الرياضي المحاكي معتمدا على طريقة التدرج من الدرجة الأولي وتطبيق أسلوب الحل باستخدام الكمبيوتر على نظام قوي حراري عند الأخذ في الاعتبار أو إهمال قدرة النقل المفقوده.
وللحصول على توليد أفضل من الناحية الاقتصادية تم تقديم أسلوب مقترح يتم فيه حساب مجموعات جديده لمعاملات صيغة قدرة النقل المفقوده وذلك تبعا للتغيرات اليوميه للحمل.
وإعادة حساب هذه المعاملات تبعا لتغير الحمل يأخذ وقت حساب أطول ولكنه يؤدي إلى تقسيم الحمل علي وحدات التوليد بدقة أكثر، وينتج عن ذلك أن تكلفة التوليد وكذلك طاقة النقل المفقوده (ميجاوات ساعه) خلال الفترة الزمنية المأخوذة في الاعتبار تكونا أقل من نظيرتيهما عند حساب المعاملات عند أول قيمه للحمل وتثبيت قيمتها بعد ذلك مع تغير الحمل.